Isogeometric Shell Analysis of the Human Abdominal Wall

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Abstract: In this paper a nonlinear isogeometric Kirchhoff-Love shell model of the human abdominal wall is proposed. Its geometry is based on *in vivo* measurements obtained from a polygon mesh that is transformed into a NURBS surface, and then used directly for the finite element analysis. The passive response of the abdominal wall model under uniform pressure is considered. A hyperelastic membrane model based on the Gasser-Ogden-Holzapfel tissue model is used together with the Koiter bending model to describe the material behavior. Due to the mixed material formulation, different sets of constitutive parameters are examined, such that the influence of each term is analyzed. The membrane contribution of the material model has a major influence on the displacement magnitude and reflects more reliably the nonlinear character of the deformation.

Keywords: abdominal wall, biomechanics, constitutive modeling, isogeometric analysis, Kirchhoff-Love shell theory.

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1 Introduction

The abdominal wall has been investigated intensively during the last two decades, especially in the context of hernia repair (Deeken and Lake, 2017). The mechanical complexity of this structure includes incompressible hyperelastic anisotropy (Gräßel et al., 2005; Astruc et al., 2018; Tran et al., 2016), active-passive muscle behavior (Grasa et al., 2016), residual stresses (Rausch and Kuhl, 2013), composite structure (Tran et al., 2014; Bielski and Lubowiecka, 2017), complex geometry with nontrivial boundary conditions (Pachera et al., 2016; Förstemann et al., 2011) and many more, including patient-specific variables, like tissue properties.

Different finite element and constitutive models have been considered so far, e.g. linear elastic orthotropic membranes (Lubowiecka et al., 2017) as well as 3D electro-mechanical continuum models for the passive and active finite strain response of muscles (Grasa et al., 2016). CT and MRI scans for detailed segmented geometry and ABAQUS® 3D hexahedral/tetrahedral finite elements for the analysis is the primary modeling approach (Pachera et al., 2016; Hernández et al., 2011; Hernández-Gascón et al., 2013; Simón-Allué et al., 2015). On the other hand, the future need of patient-specific solutions in hernia repairs and potential accessibility to the full-field *in vivo* optical measurements of the abdomen's deformation mean that efficient and computationally less expensive shell models of the abdominal wall are necessary. This coincides

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with the renaissance of rotation-free Kirchhoff-Love shell formulations in the context of Isogeometric Analysis (IGA) (Kiendl et al., 2009; Cottrell et al., 2009). The isogeometric paradigm (same NURBS functions used both for CAD modeling and FE analysis) induces high efficiency in the geometry-analysis work flow and novel refinement strategies, coherent with patient-specific applications (Morganti et al., 2015). With the use of Bézier extraction (Borden et al., 2011), isogeometric elements can be adapted to existing FE codes with no significant changes.
To the authors' knowledge, it is the first time that the abdominal wall is modelled with isogeometric shell finite elements. This approach matches the idea of patient-specific modelling and is expected be more practical than time consuming and computationally expensive 3D solid models. Additionally, a novel mixed material model, described in the following section, is used, such that the membrane and shell behavior can be distinguished.

2 A constitutive model for biological shell

The theoretical and computational shell formulation presented in Sauer and Duong (2017) and Sauer (2018) is used. The formulation is based on a fully nonlinear rotation-free Kirchhoff-Love shell model, discretized with quadratic isogeometric finite elements. The constitutive relation can be either obtained *via* projection of 3D material laws onto a two-dimensional manifold (Roohbakhshan et al., 2016), or directly derived from a 2D strain energy density function in the form

$$W = W(a_{\alpha\beta}; b_{\alpha\beta}) = W_M(a_{\alpha\beta}) + W_B(b_{\alpha\beta})\;; \tag{1}$$

where W_M is the membrane part dependent on the surface metric $a_{\alpha\beta}$ and W_B is the bending part dependent on the curvature tensor $b_{\alpha\beta}$ (Roohbakhshan and Sauer, 2017). Therefore, different stress-strain relationship can be assigned for bending and stretching separately. In this work, for a single biological shell layer with two families of embedded fibers, a mixed formulation that combines the bending energy of the 2-parameter ($_{\mathrm{K}}$; Λ) Koiter model (Steigmann, 2013) and the membrane strain energy of the incompressible 5-parameter ($_{\mathrm{GOH}}$; k_1; k_2; ;) Gasser-Ogden-Holzapfel (GOH) model (Gasser et al., 2006) is used, such that

$$\ ^{\alpha\beta} = \ _{\mathrm{GOH}}\left(A^{\alpha\beta} \quad \frac{a^{\alpha\beta}}{J^2}\right) + 2\sum_{i=1}^{2} E_i \left[\ \left(A^{\alpha\beta} \quad \frac{a^{\alpha\beta}}{J^2}\right) + (1 \quad 3\ )\, L_i^{\alpha\beta}\right]\;; \tag{2}$$

$$M_0^{\alpha\beta} = \frac{T^2}{12}\left(\Lambda\ \mathrm{tr}\mathbf{K} A^{\alpha\beta} + 2\ \ _{\mathrm{K}} K^{\alpha\beta}\right)\;; \tag{3}$$

where $^{\alpha\beta}$ are the contra-variant components of the Kirchhoff stress tensor, $M_0^{\alpha\beta}$ are the contra-variant bending moment components, $_{\mathrm{GOH/K}}$ is the 2D surface shear modulus, $\mathbf{K}$ is the relative curvature tensor with contra-variant components $K^{\alpha\beta}$, $A^{\alpha\beta}$ are the contra-variant metric components of the undeformed surface, J is the surface area change, T is the undeformed shell thickness, Λ is a 2D Lamé constant, is the fibers dispersion parameter and $L^{\alpha\beta}$ are the contra-variant components of the preferred fiber direction tensor, which is described in the local coordinates. Detailed derivations of Eqs. (2), (3) and their components can be found in Roohbakhshan et al. (2016) and Duong et al. (2017) respectively.

3 Abdominal wall analysis

3.1 Geometry of the model

The considered model is based on the geometry obtained in Szymczak et al. (2012), where the front part of the human abdominal wall was measured *in vivo*. A net of points was transformed

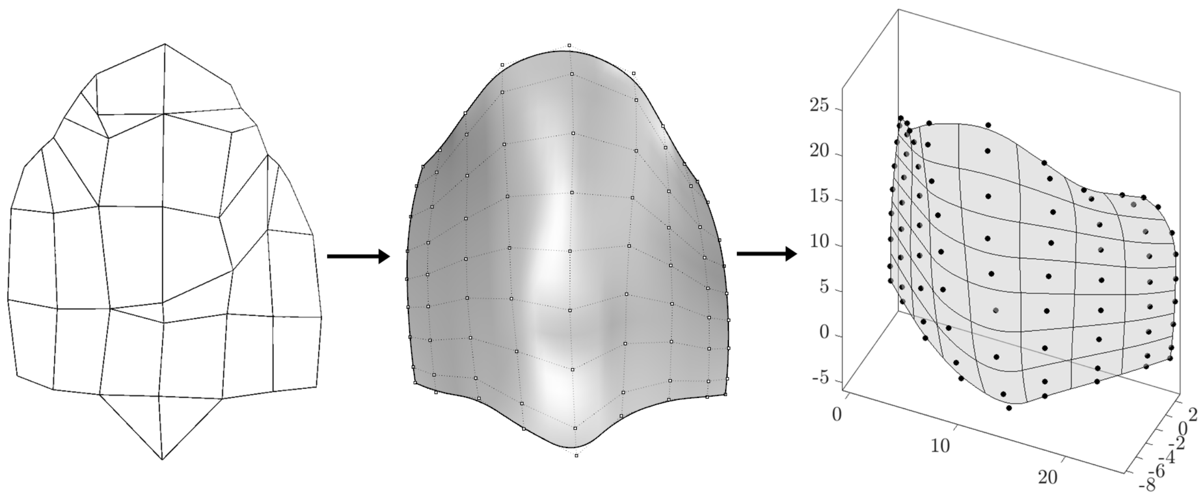

Figure 1: Geometry transfer from experimental data polygon mesh (left), *via* NURBS surface (middle) to isogeometric FE shell model (right)

into a polygon mesh, which was further modified into a single-patch NURBS surface. The control point data was then transferred into the finite element code, where the Bézier extraction procedure provided $7 \quad 7$ quadratic isogeometric finite element mesh (see Fig. 1). All boundary nodes were fixed and the load acting on the model was a deformation-following uniform pressure.

3.2 Constitutive and model parameters

Even though the GOH model has been used in abdominal wall modeling (Simón-Allué et al., 2015), its parameters are not well calibrated in these structures. Therefore, parameters that were found, though a series of analyses, to have less influence on the deformation were fixed $(T; \ ; k_2; \tilde{\Lambda})$, while the remaining parameters $\tilde{\ }_{\mathrm{GOH/K}}; \tilde{k}_1$ were chosen, such that *membrane-dominated* and *bending-dominated* behavior could be observed. The angle was chosen, such that two fiber families are orthogonal to each other ($2 \quad = 90^{\circ}$) and oriented along the *linea alba*. The thickness, from the reported range $3.0 \quad 4.5$ cm (Förstemann et al., 2011; Lubowiecka et al., 2017; Song et al., 2006), was set to a constant 3 cm, while the pressure p was set to 1600 Pa, which is the reported intra-abdominal pressure (IAP) level (12 mmHG) during laparoscopic surgery (Pachera et al., 2016; Song et al., 2006). The 3D shear modulus $\tilde{\ }_{\mathrm{GOH/K}}$ and the Lamé constant $\tilde{\Lambda}$ were calculated from linear elasticity, based on the Young modulus range found in Tran et al. (2014); Förstemann et al. (2011); Lubowiecka et al. (2017); Song et al. (2006) and Poisson's ratio $= 0.49$. The final selection of parameters is presented in Tab. 1 and sets of parameters for the analysis are collected in Tab. 2.

3.3 Results

The monitored node, the *linea alba* profile and fiber orientation are shown in Fig. (2). The finite element results for selected parameter sets are presented in Fig. (3) and Fig. (4). Maximum displacement u_{max} varies between 10-30 mm (sets 1-6, membrane dominated) 50-60 mm (sets 7-8, bending dominated) and 10-40 mm (sets 10-13, balanced). For comparison, u_{max} reported in Pachera et al. (2016) was 19.9 mm (for IAP = 2260 Pa, 3D solid model, fiber-reinforced hyperelastic material), where the numerically calculated deformation was in agreement with the evidenced one on physiological abdomens in Lubowiecka et al. (2017), i.e. $u_{max} = 16.7$ mm

Table 1: Selection of parameters

Variable	Definition	Unit	Value/range
GOH term			
$\tilde{}_{\mathrm{GOH}}$	3D shear modulus	kPa	0.5 – 10
$\tilde{k}_1$	3D stress-like parameter	kPa	1 – 10 000
k_2	fibers dimensionless parameter	–	400
	fibers dispersion parameter	–	1/3
	angle between fiber families	deg	45
Koiter term			
$\tilde{}_{\mathrm{K}}$	3D shear modulus	kPa	1 – 20
$\tilde{\Lambda}$	3D Lamé constant	kPa	330
Other			
p	IAP level	Pa	1600
T	thickness	cm	3.0

Table 2: Considered sets of parameters

No.	$\tilde{}_{\mathrm{K}}$ [kPa]	$\tilde{}_{\mathrm{GOH}}$ [kPa]	$\tilde{k}_1$ [kPa]
membrane-dominated			
set1	1	0.5	1000
set2	1	5	1000
set3	1	5	10 000
set4	1	5	100
set5	1	10	10
set6	1	10	1000
bending-dominated			
set7	8	0.5	1
set8	20	0.5	1
set9	10	1	10
balanced			
set10	8	8	10
set11	8	8	100
set12	8	8	1000
set13	8	8	10 000

(for IAP = 981 Pa, membrane model, linear ortothripic material). A direct comparison is impossible due to different IAP levels, geometry, material models and FE modeling considered by researchers. Therefore the obtained results serve as an overview of the material model capabilities in the specific application of abdominal wall modeling.

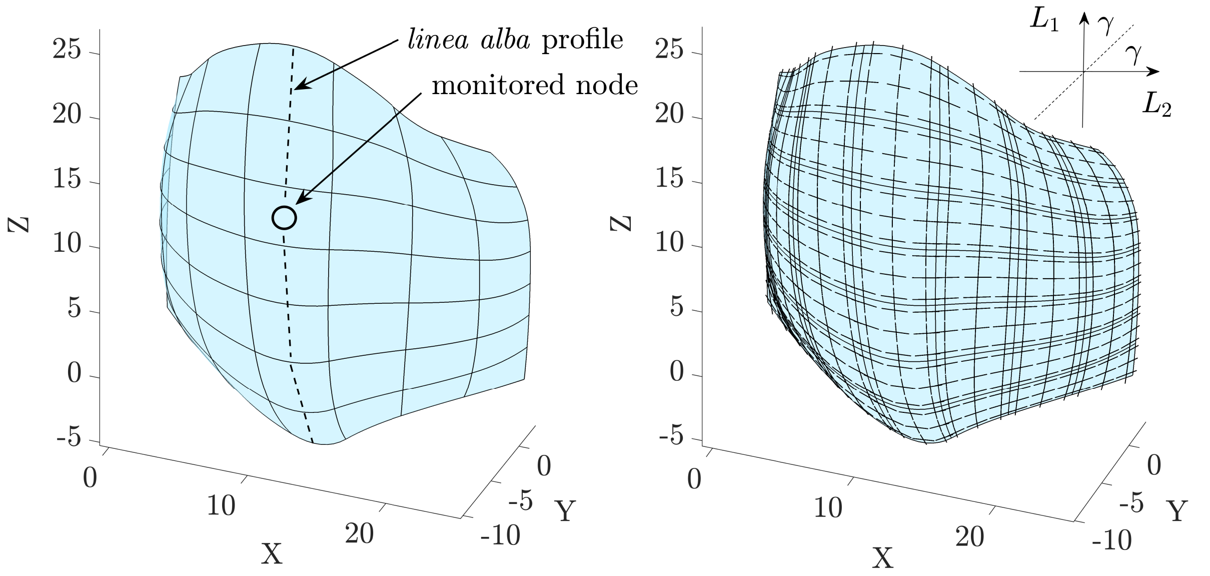

Figure 2: Deformed configuration (set4) with marked *linea alba* profile and the monitored node (*left*), with plotted fiber directions $L_1; L_2$ (*right*)

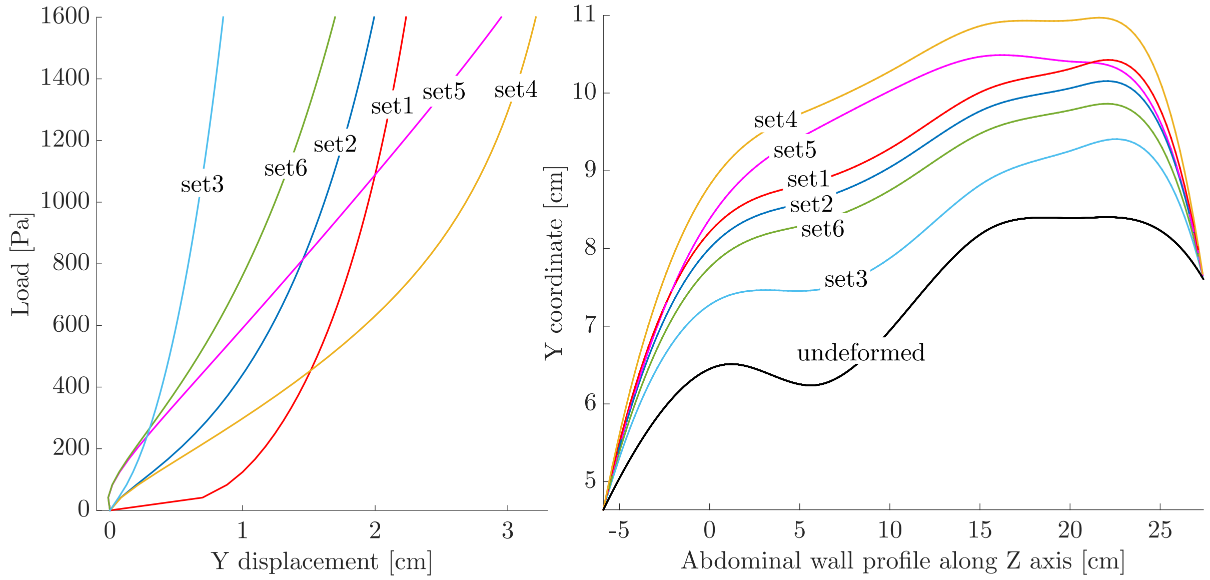

Figure 3: Load-displacement curves for the monitored node (*left*) and the displacement profile along the *linea alba* (*right*) for the membrane-dominated parameter sets

4 Discussion and conclusions

The isogeometric finite element shell model of the abdominal wall, based on *in vivo* measurements is analyzed with the use of the mixed GOH/Koiter material model. It is shown that the Koiter bending part only has a minor influence on the deformation, in comparison to the GOH membrane. It can thus be viewed merely as a membrane-stabilizing term. This is beneficial in contrast to pure membranes, where a pre-stretch is needed in order to stabilize them, which can interfere in the analysis of the residual stresses. The initial stiffness (in the linear elastic regime) is characterized by ~ (ground matrix), while the nonlinear stiffening effect is mainly characterized by $\tilde{k}_1$ (fibers), which is the typical behavior of fiber-reinforced soft tissues. In order to judge if the proposed model of the composite structure of the abdominal wall is reasonable, experimental load-displacement curves should be examined and compared with

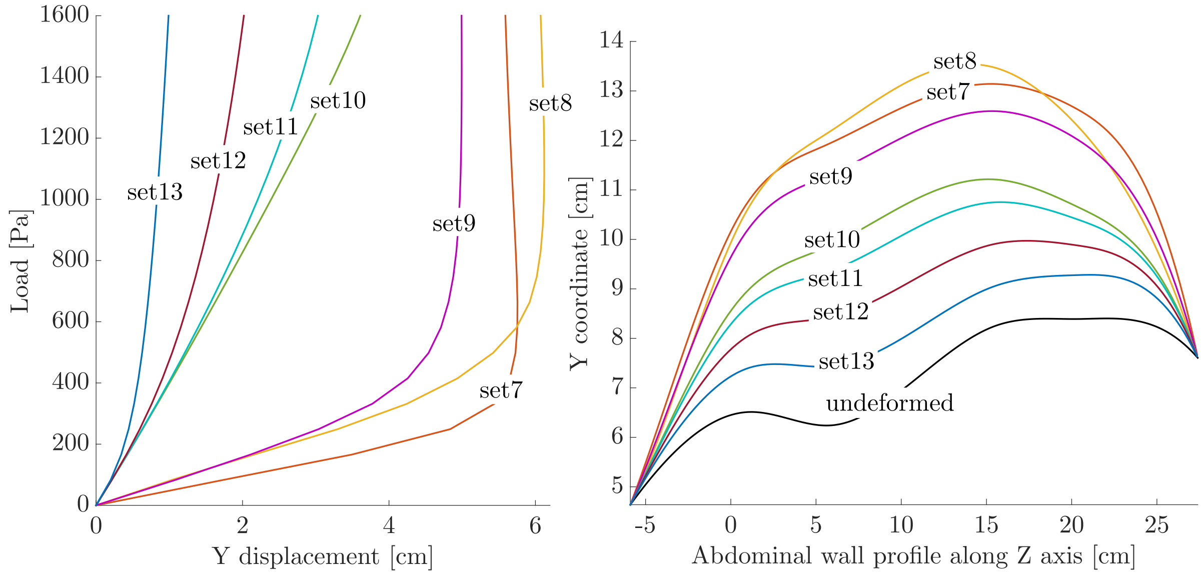

Figure 4: Load-displacement curves for the monitored node (*left*) and the displacement profile along the *linea alba* (*right*) for the bending-dominated and balanced parameter sets

computations. Also, a more detailed material model, that accounts for the different material behavior of various tissue layers, should be chosen. Future work should also focus on defining a heterogeneous distribution of the parameters and identification method, e.g. *via* an inverse analysis (Kroon and Holzapfel, 2009; Kroon, 2010).

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