A Hybridized DG / Mixed Method For Nonlinear Convection-Diffusion Problems

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Outline

Background

- HDG-BDM method for Advection-Diffusion equations.

$$\nabla \cdot (f(u) - f_v(u, \nabla u)) = 0$$

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- Non-Linear case : Proposed by J. Schütz and G. May (Promising results for N-S equations [2])

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Features of the HDG-BDM scheme

- Reduces to DG for pure advection, to BDM mixed for pure diffusion

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- It can be even mixed with the HDG scheme due to hybridization.

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BDM Mixed Method for Diffusion

Consider Laplace equation

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BDM Mixed method

$$\begin{aligned} \int_\Omega \sigma_h \cdot \boldsymbol{\tau} + \int_\Omega (\nabla \cdot \boldsymbol{\tau}) u_h - \int_{\partial\Omega} (\boldsymbol{\tau} \cdot n) g &= 0 \qquad \forall \tau \in \tilde{H}_h \\ -\int_\Omega \nabla \cdot \sigma_h \varphi &= \int_\Omega S\varphi \qquad \forall \varphi \in V_h \end{aligned}$$

Hybridizing...

The solution spaces : $u_h \in V_h$, $\sigma_h \in H_h$, $\lambda_h \in M_h$

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Hyb. BDM mixed method

$$
\begin{aligned}
\sum_k \int_{\Omega_k} \sigma_h \cdot \boldsymbol{\tau} + \int_{\Omega_k} (\nabla \cdot \boldsymbol{\tau}) u_h - \int_{\partial\Omega_k} (\boldsymbol{\tau} \cdot n) \lambda_h &= 0 && \forall \tau \in H_h \\
-\sum_k \int_{\Omega_k} (\nabla \cdot \sigma_h) \varphi &= \sum_k \int_{\Omega_k} S\varphi && \forall \varphi \in V_h \\
\sum_k \int_{\partial\Omega_k} -(\sigma_h \cdot n)\mu &= 0 && \forall \mu \in M_h
\end{aligned}
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Adding DG for Advection

Advection-Diffusion equation

$$\nabla \cdot f(u) - \epsilon \nabla \cdot \nabla u = S$$

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HDG-BDM method

$$\begin{aligned}
\sum_k \int_{\Omega_k} \epsilon^{-1} \sigma_h \cdot \boldsymbol{\tau} + \int_{\Omega_k} (\nabla \cdot \boldsymbol{\tau}) u_h - \int_{\partial\Omega_k} (\boldsymbol{\tau} \cdot n) \lambda_h &= 0 \\
\sum_k \int_{\Omega_k} -f(u_h) \cdot \nabla \varphi + \int_{\Gamma_k} \varphi \left(f(\lambda_h) \cdot n - \alpha(\lambda_h - u_h) \right) - \int_{\Omega_k} (\nabla \cdot \sigma_h) \varphi & \\
= \sum_k \int_{\Omega_k} S \varphi & \\
\sum_k \int_{\partial\Omega_k} (-\sigma_h \cdot n + f(\lambda_h) \cdot n - \alpha(\lambda_h - u_h)) \mu &= 0
\end{aligned}$$

Proposed by Nguyen et. al [5]

The solution spaces : $u_h \in \tilde{V}_h$, $\sigma_h \in H_h$, $\lambda_h \in M_h$

$$\tilde{V}_h \quad := \quad \{\varphi \in L^2(\Omega) : \quad \varphi|_{\Omega_k} \in P^m(\Omega_k)\}$$

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HDG method

$$\sum_k \int_{\Omega_k} \epsilon^{-1}\sigma_h \cdot \boldsymbol{\tau} + \int_{\Omega_k} (\nabla \cdot \boldsymbol{\tau}) u_h - \int_{\partial\Omega_k} (\boldsymbol{\tau} \cdot n)\lambda_h \quad = \quad 0$$

$$\sum_k \int_{\Omega_k} -f(u_h) \cdot \nabla\varphi + \int_{\Gamma_k} \varphi\left(f(\lambda_h) \cdot n - \beta(\lambda_h - u_h)\right) - \int_{\Omega_k} (\nabla \cdot \sigma_h)\varphi = \sum_k \int_{\Omega_k} S\varphi$$

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Comparison

HDG-BDM

$$u_h|_{\Omega_k} \in P^{m-1}$$

$$-\sigma_h + \hat{f}_h = -\sigma_h + f(\lambda_h) - \alpha(\lambda_h - u_h)\mathbf{n}$$

HDG

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HDG

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Common method

$$\sum_k \int_{\Omega_k} \epsilon^{-1}\sigma_h \cdot \boldsymbol{\tau} + \int_{\Omega_k} (\nabla \cdot \boldsymbol{\tau}) u_h - \int_{\partial\Omega_k} (\boldsymbol{\tau} \cdot n)\lambda_h = 0$$

$$\sum_k \int_{\Omega_k} -f(u_h) \cdot \nabla\varphi + \int_{\Gamma_k} \varphi\left(f(\lambda_h) \cdot n - (\alpha|\beta)(\lambda_h - u_h)\right) - \int_{\Omega_k} (\nabla \cdot \sigma_h)\varphi = \sum_k \int_{\Omega_k} S\varphi$$

$$\sum_k \int_{\partial\Omega_k} \left(-\sigma_h \cdot n + f(\lambda_h) \cdot n - (\alpha|\beta)(\lambda_h - u_h)\right)\mu = 0$$

ces

Convergence

- Post processing of the solution [6]

	HDG-RT	HDG-BDM	HDG [7]	Post Proc. Conv.
u_h	P^m	P^{m-1}	P^m	$m+2$
σ_h	RT^m	P^m	P^m	$m+1$

[6] R. Stenberg, Math. Model. Numer. Anal. 25. 151-168, 1991

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- For HDG-BDM and HDG-RT[8] , this optimal condition is when diffusion dominates and one can put $\alpha = 0$

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Two dimensional viscous Burgers equation

$$\begin{aligned} \frac{1}{2}\nabla \cdot (u^2, u^2) - \epsilon \nabla \cdot \nabla u &= S \quad in \ \Omega \\ u &= 0 \quad in \ \partial\Omega \end{aligned}$$

Solution :

$$u(x,y) = \left(x + \frac{e^{c_1 x/\epsilon} - 1}{1 - e^{c_1/\epsilon}} \right) \cdot \left(y + \frac{e^{c_1 y/\epsilon} - 1}{1 - e^{c_1/\epsilon}} \right)$$

Test case 1 : Boundary Layer

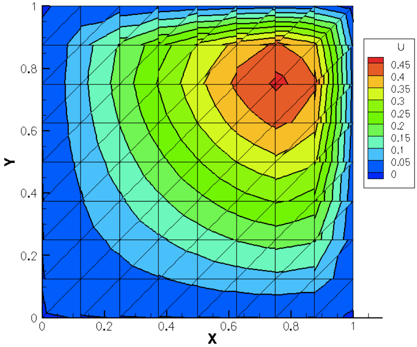

Figure: Contours of u, $m = 2$ ($u \in P^1$), $\epsilon = 0.1$, HDG-BDM scheme

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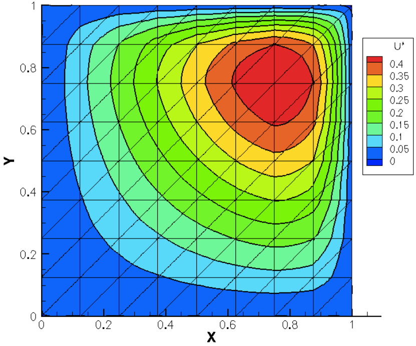

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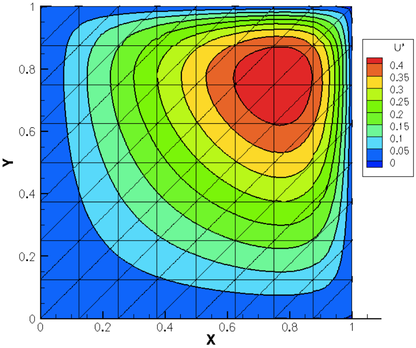

Figure: Contours of u^*, $m = 2$ $(u \in P^1)$, $\epsilon = 0.1$, $\alpha = 0$, HDG-BDM scheme

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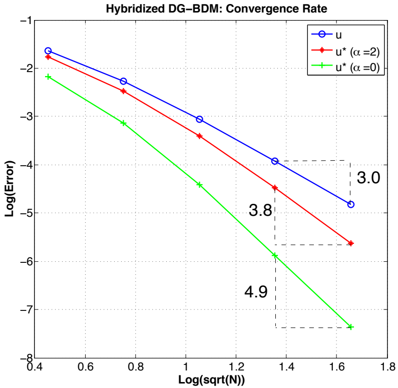

Figure: Convergence, $m = 3$ ($u \in P^2$), $\epsilon = 0.1$, HDG-BDM scheme

Test case 2 : Linear Boundary Layer

Mixing HDG and HDG-BDM methods :

Condition: If Peclect number, $Pe = \frac{|c|h}{\epsilon} < 5$, then use HDG-BDM

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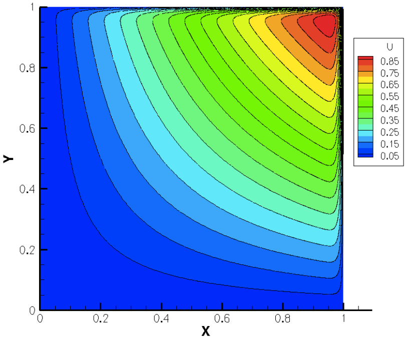
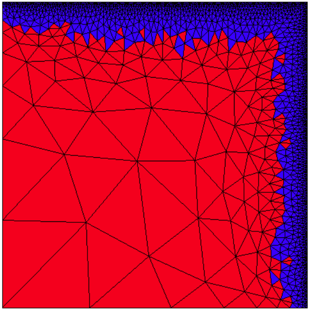
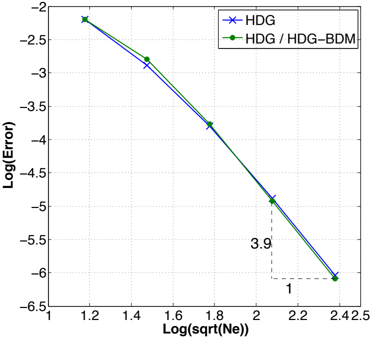


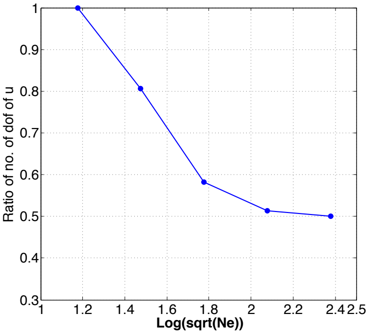
Contours of u^*, $m = 2$, $\epsilon = 0.01$



Red : HDG, Blue : HDG-BDM



Convergence of u^*, $m = 2$



Reduction of dofs of u

Test case 3

Advection Diffusion equation:

$$\begin{aligned} \nabla \cdot u - \nabla \cdot (\epsilon(x)\nabla u) &= S \qquad in \;\; \Omega \\ u &= g \qquad in \;\; \Gamma_D \end{aligned}$$

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Diffusion Coefficient:

$$\epsilon = \begin{cases} 0.001, & x \leq 0.9 \\ 1, & x \geq 1.1 \\ smooth\ fn., & 0.9 < x < 1.1 \end{cases}$$

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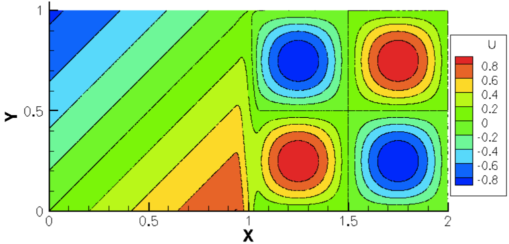
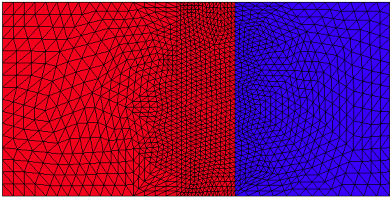
Solution:

$$u(x, y) = (1 - \epsilon(x)) \sin(x - y) + \epsilon(x) \sin(2\pi x) \sin(2\pi y)$$



Condition : $x > 1.2$, use HDG-BDM



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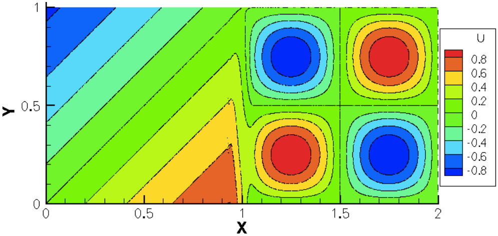

Figure: Contours of u^*, $m = 2$

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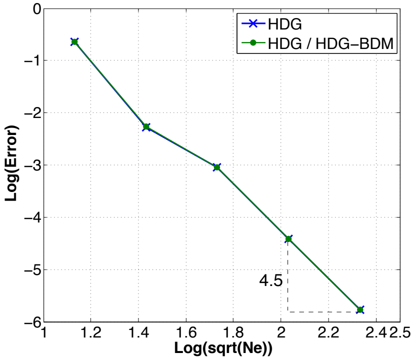

Figure: Convergence of u^*, $m = 2$

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Condition : $Pe < 5$, use HDG-BDM

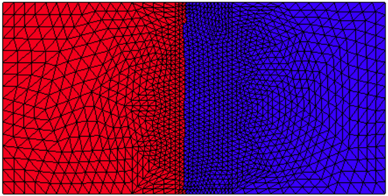

Figure: Red : HDG, Blue : HDG-BDM

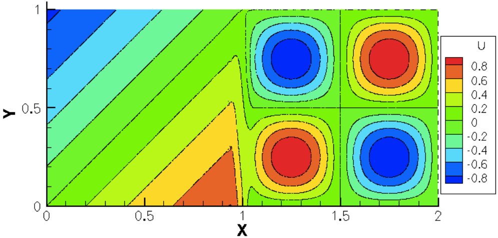

Figure: Contours of u^*, $m = 2$

Condition : $Pe < 5$, use HDG-BDM

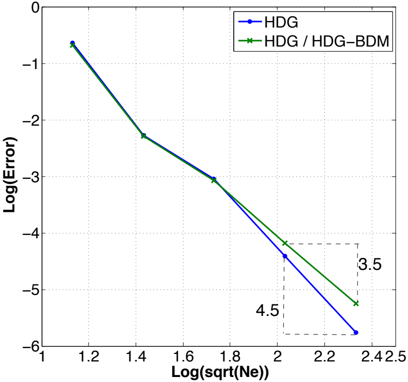

Figure: Convergence of u^*, $m = 2$

Conclusions

Present work :

- HDG-BDM method and it's connection with HDG scheme

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Acknowledgement

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Deutsche Forschungsgemeinschaft

Cell-wise discretization of the Neumann problem :

$$\begin{aligned} \epsilon(\nabla u_h^*, \nabla \phi) &= (\sigma_h, \nabla \phi) \qquad \forall \phi \in P_0^q(\Omega_k) \\ (u_h, 1) &= (u_h^*, 1) \end{aligned}$$

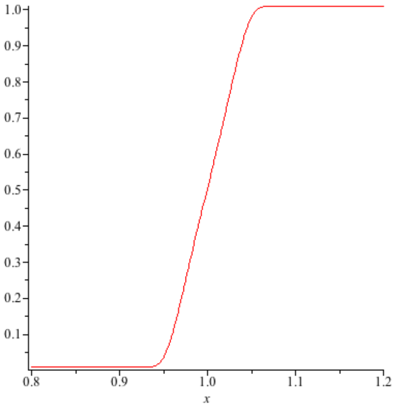
where

$$P_0^q(\Omega_k) := \{\phi \in P^q(\Omega_k), (\phi, 1) = 0\}$$

with $q = m + 1$ for HDG and HDG-BDM ($\alpha = 0$) and $q = m$ for HDG-BDM ($\alpha \neq 0$).

Smooth Function

$$\epsilon = e^{(-9+10x)^{-2}}\left(e^{(-9+10x)^{-2}} + e^{(-11+10x)^{-2}}\right)^{-1}$$



Test case 1 : Boundary Layer

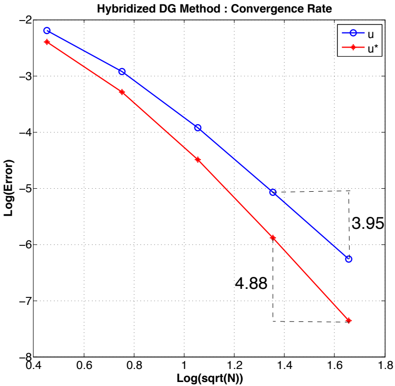


Figure: Convergence, $m = 3$ ($u \in P^3$), $\epsilon = 0.1$, HDG scheme

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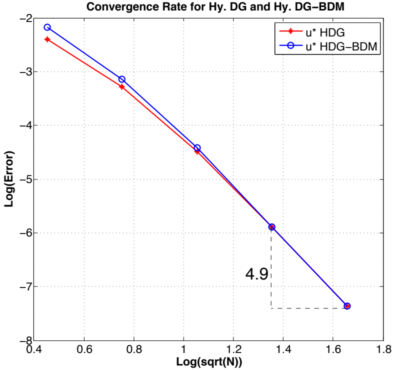

Figure: Convergence, $m = 3$, $\epsilon = 0.1$, HDG ($u \in P^3$) and HDG-BDM ($u \in P^2$) schemes