

Quasi-Static Analysis of Self-Cleaning Surface Mechanisms

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Abstract

Based on a finite element (FE) model, we study the self-cleaning effect, also known as the lotus effect, which is observed on hydrophobic surfaces. The interaction between these surfaces, liquid droplets, and pollutant particles is investigated through a force analysis. Some forces such as the capillary force and the contact line force, require the computation of the liquid droplet membrane deformation, governed by the Young-Laplace equation. Based on this analysis, we compute the net force governing the behaviour of a particle in contact with a 2D liquid droplet. This work provides answers to the following questions: In a quasi-static framework, does the self-cleaning mechanism work for given surface and droplet parameters? I.e: would a particle be lifted off by the water droplet or not? How do the model parameters affect the net force acting on the particle? The parameters considered in this study are: volume and density of the liquid droplet, and size, density and contact angle of the pollutant particle.

Keywords: Self-cleaning mechanism, contact angle, static wetting, nonlinear finite element analysis, droplet membranes.

1 Introduction

The self-cleaning phenomenon, also called the lotus effect, is observed in some natural and artificial surfaces. These are hydrophobic surfaces which minimize the surface contact with water, thus splitting it into small spherical droplets. This allows a smooth rolling/sliding over the surface while sweeping away pollutant particles. The mechanical principles behind this mechanism are complex as they involve the coupling of several problems; contact on multi-scale rough surfaces, fluid flow inside the droplets, droplet membrane deformation, wettability and contact angle, and the interaction between droplet and pollutant particles. In this work, we discuss some aspects of the last three problems, and provide solutions based on FE computations.

Liquid droplets can be treated computationally as a structural membrane governed by the Young-Laplace equation, and an internal liquid flow governed by the Stokes equation. Different approaches can be used to solve the two problems. A simple approach is solving the two problems in a decoupled manner, where each problem is solved separately [Osman et al. \(2013\)](#). For quasi-static droplets, an internal bulk pressure substitutes the flow, as in Sauer et al. [Sauer et al. \(2014\)](#) and Sauer [Sauer \(2014\)](#), where stabilized formulations are presented for modelling

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of liquid membranes in static contact, based on the finite element method (FEM). Osman et al. [Osman et al. \(2013\)](#) studies dynamic contact of droplets on rough surfaces, considering Stokes model for the internal liquid flow. The interaction between solid particles and air bubbles inside liquids is investigated by Schulze [Schulze \(1984\)](#). Osman et al. [Osman and Sauer \(2010\)](#) and Kralchevsky et al. [Kralchevsky and Nagayama \(2001\)](#) discuss the force analysis involved in attachment/detachment of small pollutant particles to/from large liquid droplets under the assumption of a pre-defined location of the contact line as a boundary condition, and flat non-deformable liquid surface. Here we extend this study to provide solutions for arbitrary sizes of droplets and particles. Furthermore, the contact line is obtained from the numerical solution of the deformed liquid membrane.

2 FE Model

2.1 Governing equations

A system of a quasi-static liquid droplet, a flat rigid surface and a spherical pollutant particle is considered. The difference between the internal and the external pressure on the liquid membrane interface Δp is balanced by the surface curvature $2H$, through the Young-Laplace equation,

$$2H\gamma_{LG} = \Delta p \quad [N/m^2], \quad (1)$$

where γ_{LG} is the surface tension at the gas-liquid interface. The pressure difference across the interface can be expressed as

$$\Delta p = p_f - p_c, \quad p_f = p_0 + \rho_w g y, \quad (2)$$

where p_f is the fluid bulk pressure comprising the capillary pressure p_0 and the hydrostatic pressure in terms of the liquid density ρ_w , gravity g and the surface height y . A contact pressure p_c appears where interactions between the membrane and other surfaces take place. For normalization, we multiply Eq.(1) by L/γ_{LG} to obtain the dimensionless quantities marked with tilde,

$$2\tilde{H} = \tilde{p}_f - \tilde{p}_c, \quad \tilde{p}_f = \lambda + B \tilde{y}, \quad B = \frac{\rho_w g L^2}{\gamma_{LG}}, \quad (3)$$

where λ is the Lagrange multiplier accounting for the capillary pressure, B is the so called Bond number, and L is the characteristic length, usually taken as the droplet diameter. The contact line \mathcal{L}_c is the location where the three phases co-exist, forming a specific contact angle θ with the particle surface, denoted as θ_p (θ_s in case of contact with the substrate surface). The force equilibrium at the contact line is expressed in terms of the interfacial tractions \mathbf{t}_{SG} , \mathbf{t}_{LG} and \mathbf{t}_{SL} at the solid-gas, liquid-gas and solid-liquid interfaces, respectively, as,

$$\mathbf{t}_{SG} + \mathbf{t}_{LG} + \mathbf{t}_{SL} + \mathbf{q}_n = \mathbf{0}, \quad (4)$$

where \mathbf{q}_n is the force which counterbalances the normal projection of \mathbf{t}_{LG} w.r.t the substrate surface (particle). The normal and tangential components of Eq.(4), respectively read,

$$\gamma_{SG} - \gamma_{LG} \cos \theta - \gamma_{SL} = 0, \quad (5)$$

$$q_n - \gamma_{LG} \sin \theta = 0, \quad (6)$$

where γ_{SL} , γ_{SG} and γ_{LG} denote the surface tension at the three interfaces. Eq.(5) is known as Young's equation. The FE implementation of the above equations can be found in [Sauer \(2014\)](#).

2.2 Force balance

The considered spherical pollutant particle of radius r_p and density ρ_p , is subjected to four forces shown in Fig.(1): particle weight \mathbf{F}_G , contact line force \mathbf{F}_{CL} , hydrostatic force \mathbf{F}_H , and buoyancy force \mathbf{F}_B , defined as follows:

$$\mathbf{F}_G = \frac{4}{3}\pi r_p^3 \rho_p \mathbf{g}, \quad (7)$$

$$\mathbf{F}_{CL} = \oint_{\mathcal{L}_c} \mathbf{t}_{LG} d\mathcal{L}_c, \quad (8)$$

$$\mathbf{F}_H = \int_{a_w} p_f \mathbf{n} da_w \approx p_0 A_w \mathbf{N}, \quad (9)$$

$$\mathbf{F}_B = \rho_w g \mathcal{V}_w \mathbf{N}, \quad \mathcal{V}_w = \frac{\pi b}{6}(3a^2 + b^2), \quad (10)$$

where \mathbf{n} is the normal to the wetted area A_w , while \mathcal{V}_w is the wetted volume of the particle, \mathbf{N} is the normal to the contact line along the particle axis, and a & b are distances defined in Fig.(1). The effective force \mathbf{F}_e is the summation of all forces. Among the above parameters, the location of the contact line \mathcal{L}_c , the traction along the liquid-gas interface \mathbf{t}_{LG} , and the internal pressure p_f require computation of the membrane deformation. Friction and surface adhesion between the particle and the substrate are not considered in this work.

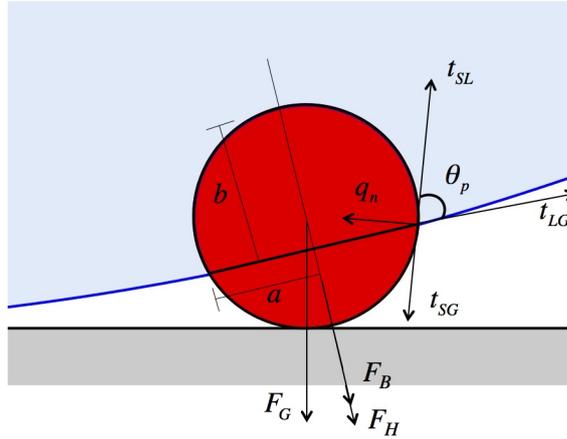


Figure 1: Schematic of the forces acting on a particle in contact with a liquid droplet.

3 Results and Discussion

Computations here are based on the stabilized FE formulation for liquid membranes [Sauer \(2014\)](#). In the following examples, we consider quasi-static droplets in contact with a superhydrophobic flat surface ($\theta_s = 180^\circ$) and a spherical pollutant particle with various contact angle ($\theta_p = 30^\circ - 180^\circ$). The particle is considered fixed to the substrate surface at different locations, and the membrane deformation is computed for different parameters such as the volume and density of the droplet and the particle, as well as θ_p . Knowing the location of the contact line and the associated membrane surface tangents, the force balance can be obtained, and thus the effective force \mathbf{F}_e . The vertical component of \mathbf{F}_e determines whether the particle is lifted to the droplet or not.

3.1 Example 1

We consider a weightless spherical droplet ($B = 0$), of radius $R = 2L$, with Dirichlet boundary conditions at the upper half, in contact with a spherical rigid pollutant particle fixed at three different locations along the vertical axis of symmetry of the droplet. These locations are distinguished by the vertical distance y_p , measured from the centre of the particle to the horizontal level (marked with dotted line in Fig.(2)), which is tangent to the undeformed droplet from the bottom. Although the contact angle of the particle is fixed to $\theta_p = 120^\circ$, the direction of the effective force \mathbf{F}_e alters as the particle penetrates into the droplet (i.e y_p increases), as shown in Fig.(2). This is due to the membrane deformation, which causes a change in the direction of the contact line force, which is dominant in the case of relatively light particles ($\rho_p = \rho_w$).

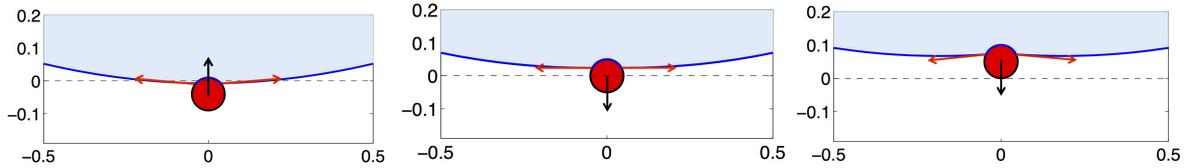


Figure 2: FE solution of a weightless droplet ($B=0$) in contact with a pollutant particle of contact angle $\theta_p = 120^\circ$ and radius $r_p = 0.05L$, fixed at a distance $y_p = -0.025, 0$, and $0.05L$ (left to right). Black arrows show the direction of the effective force \mathbf{F}_e .

3.2 Example 2

In the second example we consider an axisymmetric water droplet ($B = 0.1316$, $R = 2L$) on a flat surface with $\theta_s = 180^\circ$, and in contact with a spherical rigid pollutant particle ($\rho_p = \rho_w$, $r_p = 0.05L$) fixed at a point on the substrate surface where the droplet membrane is just touching the particle at $\theta_p = 180^\circ$ (see Fig.(3)). Dirichlet boundary conditions in the horizontal direction are applied to the droplet membrane at the axis of symmetry. The droplet deformation is shown for the range of $\theta_p = 30^\circ - 180^\circ$ in Fig.(5). The particle is lifted towards the droplet when the sign of $\mathbf{F}_e \cdot \mathbf{g}$ is negative, which means \mathbf{F}_e points upwards, at a critical θ_p . A smaller particle is lifted to the droplet at a higher contact angle, as observed in Fig.(4).

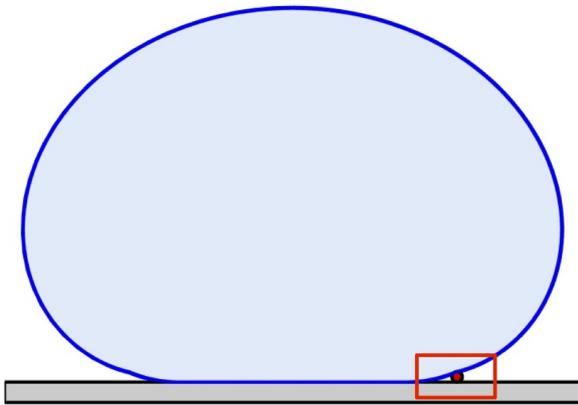


Figure 3: FE solution for a droplet in contact with a flat surface ($\theta_s = 180^\circ$) and a spherical rigid particle with $\theta_p = 180^\circ$.

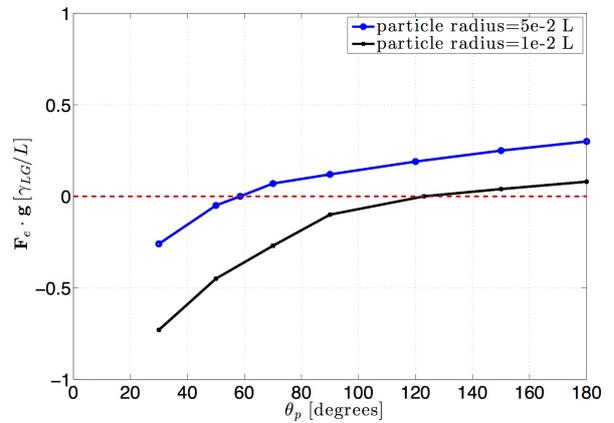


Figure 4: Equilibrium force at various contact angles θ_p of the pollutant particle.

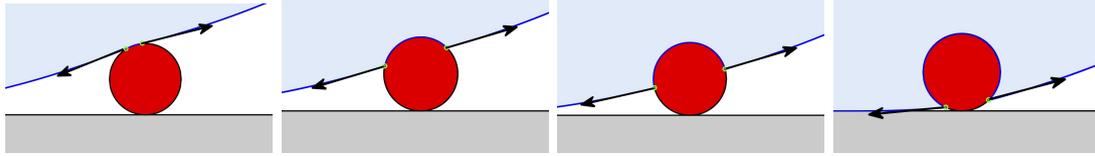


Figure 5: FE solution of a deformed water droplet ($B=0.1316$) in contact with a flat surface, and a pollutant particle ($r_p = 0.05L$) of contact angle $\theta_p = 180^\circ, 120^\circ, 90^\circ$ and 30° (left to right). Black arrows show the direction of the tangential contact line force.

4 Conclusions

A force analysis for the interaction of pollutant particles with quasi-static liquid droplets is presented. A numerical treatment of the droplet deformation is required to determine the contact line force. A particle is lifted up to the droplet if the contact line force is large enough to overcome the other forces. The direction of the contact line force depends on the contact angle and size of the particle, membrane deformation at the contact line, the droplet volume, and the penetration of the particle into the droplet. The first example showed that a detachment is possible if the particle is further penetrated, for example by other forces, into the droplet. The second example showed that smaller particles are lifted at higher contact angles, compared to larger particles.

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References

- Kralchevsky, P. and Nagayama, K. (2001). *Particles at Fluids Interfaces and Membranes: Attachment of Colloid Particles and Proteins to Interfaces and Formation of Two-Dimensional Arrays*. Elsevier, Vol. 10.
- Osman, M., Rasool, R., and Sauer, R. A. (2013). Computational aspects of self-cleaning surface mechanisms. In Mittal, K., editor, *Advances in Contact Angle, Wettability and Adhesion*. Edited by K.L. Mittal, chapter 7. Scrivener.
- Osman, M. and Sauer, R. (2010). Mechanical modeling of particle-droplet interaction motivated by the study of self-cleaning mechanisms. *Proc. Appl. Math. Mech.*, **10**:85–86.
- Sauer, R. A. (2014). Stabilized finite element formulations for liquid membranes and their application to droplet contact. *Int. J. Numer. Meth. Fluids*, page DOI: 10.1002/fld.3905.
- Sauer, R. A., Duong, X. T., and Corbett, C. (2014). A computational formulation for constrained solid and liquid membranes considering isogeometric finite elements. *Comput Method Appl Mech Engrg*, 271:pp. 48–68.
- Schulze, H. (1984). *Physico-chemical Elementary Processes in Flotation*. Elsevier.